

Show **all** of your work on this assignment and answer each question fully in the given context.

Please staple your assignment!

1. Suppose that  $X$  is a random variable with probability density function

$$f(x) = \begin{cases} c \cdot x^2 & -2 \leq x \leq 2 \\ 0 & \text{o.w.} \end{cases}$$

- (a) Find the value of  $c$  that makes  $f(x)$  a valid probability density function. [5 pts]  
 (b) Find the CDF of the random variable  $X$ . [5 pts]  
 (c) What is  $P(|X| \geq -1)$  [5 pts].  
 (d) Find the expected value of  $X$ . [5 pts]
2. Consider a continuously distributed random variable,  $W$ , with a probability density function given by

$$f(w) = \begin{cases} \frac{1}{5(1-e^{-2})} e^{-w/5} & 0 \leq w \leq 10 \\ 0 & \text{otherwise} \end{cases}$$

- (a) Show that the function  $f(w)$  is a valid probability density function (i.e., show that (i)  $f(w)$  is non-negative and (ii)  $\int_{-\infty}^{\infty} f(w)dw = 1$ ). [5 pts]  
 (b) Find  $P(W \leq 2)$  [5 pts]  
 (c) Find  $P(2 \leq W \leq 10)$  [5 pts]  
 (d) Find  $E(W)$  [5 pts]
3. The mileage to first failure for a model of military personnel carrier can be modeled as exponential with mean 1,000 miles.
- (a) Find the probability that a vehicle of this type gives less than 500 miles of service before first failure. [5 pts]  
 (b) Find the probability that a vehicle of this type gives less than 2000 miles of service before first failure. [5 pts]
4. (Ch. 5.2, Exercise 2, pg. 263) Suppose that  $Z$  is a standard normal random variable. Evaluate the following probabilities involving  $Z$ :

- (a)  $P[Z < -.62]$  [3 pts]  
 (b)  $P[Z > 1.06]$  [3 pts]  
 (c)  $P[-.37 < Z < .51]$  [3 pts]  
 (d)  $P[|Z| \leq .47]$  [3 pts]  
 (e)  $P[|Z| > .93]$  [3 pts]  
 (f)  $P[-3.0 < Z < 3.0]$  [3 pts]

Now find numbers # such that the following statements involving  $Z$  are true:

- (a)  $P[Z \leq \#] = .90$  [3 pts]  
 (b)  $P[|Z| \leq \#] = .90$  [3 pts]  
 (c)  $P[|Z| > \#] = .03$  [3 pts]

Total: 77 pts