STAT 305: Chapter 5

Part III

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Continuous Random Variables

Terminology, Use, and Common Distributions

What is a Continuous Random Variable?

Background on Continuous Random Variable

Along with discrete random variables, we have continuous random variables. While discrete random variables take one specific values from a *discrete* (aka countable) set of possible real-number values, continous random variables take values over intervals of real numbers.

> **def: Continuous random variable**
> A continuous random variable is a random variable which takes values on a continuous interval of real numbers.

The reason we treat them differently has mainly to do with the differences in how the math behaves: now that we are dealing with interval ranges, we change summations to integrals.

Background

What?

Examples of continuous random variable:

> **Z** is the amount of torque required to lossen the next bold (not rounded)
>
> **T** is the time you will wait for the next bus
>
> **C** is the outside temprature at 11:49 pm tomorrow
>
> **L** is the length of the next manufactured metal bar
>
> **V** is the yield of the next run of process

Terminology and Usage

Background

Terminology

pdf

Probability Density Function

Since we are now taking values over an interval, we can not "add up" probabilities with our probability function anymore. Instead, we need a new function to describe probability:

> **def: probability density function**
> A probability density function (pdf) defines the way the probability of a continuous random variable is distributed across the interval of values it can take. Since it represents probability, the probability function must always be non-negative. Regions of higher density have higher probability.

Background

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Probability Density Function

Validity of a *pdf*

Any function that satisfies the following can be a probability density function:

1. $\int_{-\infty}^{\infty} f(x)dx = 1$
2. $f(x) \geq 0$ for all x in $(-\infty, \infty)$

and such that for all $a \leq b$,

$$P(a \leq X \leq b) = P(a \leq X < b) = P(a < X \leq b) = P(a < X < b)$$

$$= \int_a^b f(x)dx.$$

Background

Terms and Use

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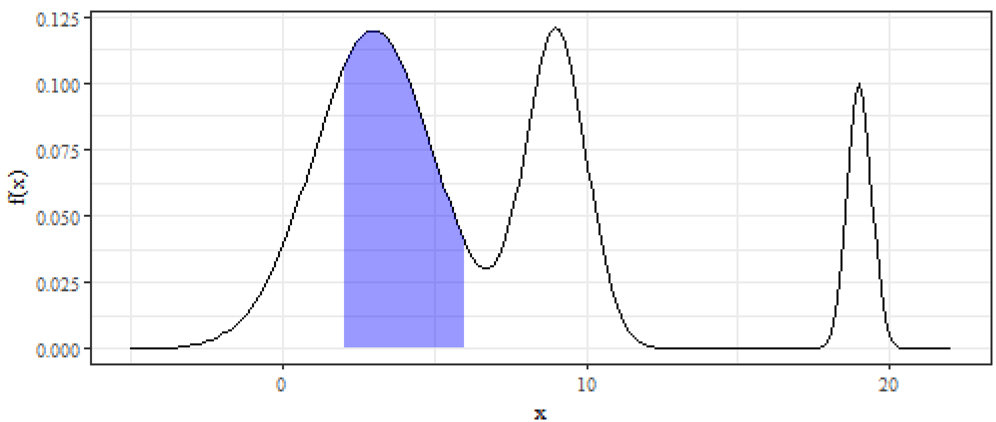
Probability Density Function

With continuous random variables, we use pdfs to get probabilities as follows:

> For a continuous random variable X with probability density function $f(x)$,
>
> $$P(a \le X \le b) = \int_a^b f(x)dx$$
>
> `for any real values`(a, b)`such that`(a\le b)`



Terms and Use

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Example

Consider a de-magnetized compass needle mounted at its center so that it can spin freely. It is spun clockwise and when it comes to rest the angle, θ, from the vertical, is measured. Let

$$Y = \text{the angle measured after each spin in radians}$$

What values can Y take?

What form makes sense for $f(y)$?

Terms and Use

pdf

Example

If this form is adopted, that what must the pdf be?

Using this pdf, calculate the following probabilities:

- $P[Y < \frac{\pi}{2}]$

Example

Terms and Use

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- $P[\frac{\pi}{2} < Y < 2\pi]$

- $P[Y = \frac{\pi}{6}]$

Cumulative Density Function (**CDF**)

We also have the cumulative density function for continuous random variables:

> **def: Cumulative density function (cdf)** For a continous random variable, X, with pdf f(x) the cumulative density function $F(x)$ is defined as the probability that X takes a value less than or equal to x which is to say
>
> $$F(x) = P(X \leq x) = \int_{-\infty}^{x} f(t)dt$$

TRUE FACT: the Fundamental Theorem of Calculus applies here:

$$\frac{d}{dx}F(x) = f(x)$$

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Cumulative Density Function (**CDF**)

Properties of CDF for continuous random variables

As with discrete random variables, F has the following properties:

- **F** is monotonically increasing (i.e it is never decreasing)
- $\lim_{x \to -\infty} F(x) = 0$ and $\lim_{x \to +\infty} F(x) = 1$
  - This means that $0 \le F(x) \le 1$ for **any CDF**
- **F** is *continuous*. (instead of just right continuous in discrete form)

Mean and Variance

of

Continuous Random Variables

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E(X), V(X)

Expected Value and Variance

Expected Value

As with discrete random variables, continuous random variables have expected values and variances:

> **def: Expected Value of Continuous Random Variable**
> For a continous random variable, X, with pdf f(x) the expected value (also known as the mean) is defined as
>
> $$E(X) = \int_{-\infty}^{\infty} x f(x) dx$$

We often use the symbol μ for the mean of a random variable, since writing $E(X)$ can get confusing when lots of other parenthesis are around. We also sometimes write EX.

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E(X), V(X)

Expected Value and Variance

Variance

> **def: Variance of Continuous Random Variable**
> For a continous random variable, X, with pdf f(x) and expected value μ, the variance is defined as
>
> $$V(X) = \int_{-\infty}^{\infty} (x-\mu)^2 \cdot f(x)dx$$
>
> which is identical to saying
>
> $$V(X) = E(X^2) - E(X)^2$$

We will sometimes use the symbol σ^2 to refer to the variance and you may see the notation $VarX$ or VX as well.

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E(X), V(X)

Expected Value and Variance

Sdandard Deviation (SD)

We can also use the variance to get the standard deviation of the random variable:

> **def: Standard Deviation of Continuous Random Variable**
> For a continous random variable, X, with pdf f(x) and expected value μ, the standard deviation is defined as:
>
> $$\sigma = \sqrt{\sigma^2} = \sqrt{\int_{-\infty}^{\infty} (x-\mu)^2 \cdot f(x)dx}$$

Expected Value and Variance: Example

Library books

Let X denote the amount of time for which a book on 2-hour hold reserve at a college library is checked out by a randomly selected student and suppose its density function is

$$f(x) = \begin{cases} 0.5x & 0 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

Calculate $\mathrm{E}X$ and $\mathrm{Var}X$.

An important point about Expected Value and Variance of Random Variables

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Expected Value and Variance:

For a linear function, $g(X) = aX + b$, where a and b are constants,

> $$\mathrm{E}(aX+b) = a\mathrm{E}(X) + b$$
>
> $$\mathrm{Var}(aX+b) = a^2\mathrm{Var}(X)$$

e.g Let $X \sim Binomial(5, 0.2)$. What is the expected value and variance of 4X- 3?

Common Distributions

Uniform Distribution

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Common continuous Distributions

Uniform Distribution

For cases where we only know/believe/assume that a value will be between two numbers but know/believe/assume *nothing* else.

Origin: We know a the random variable will take a value inside a certain range, but we don't have any belief that one part of that range is more likely than another part of that range.

> **Definition: Uniform random variable**
> The random variable U is a uniform random variable on the interval $[a, b]$ if it's density is constant on $[a, b]$ and the probability it takes a value outside $[a, b]$ is 0. We say that U follows a uniform distribution or $U \sim uniform(a, b)$.

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Uniform Distribution

> **Definition: Uniform pdf**
> If U is a uniform random variable on $[a, b]$ then the probability density function of U is given by
>
> $$f(u) = \begin{cases} \dfrac{1}{b-a} & a \leq u \leq b \\ 0 & o.w. \end{cases}$$
>
> `

With this, we can find the for any value of a and b, if $U \sim uniform(a, b)$ the mean and variance are:

$$E(U) = \frac{1}{2}(b-a)$$

$$Var(U) = \frac{1}{12}(b-a)^2$$

Uniform Distribution

Definition: Uniform cdf
If U is a uniform random variable on $[a, b]$ then the cumulative density function of U is given by

$$F(u) = \begin{cases} 0 & u < a \\ \dfrac{u-a}{b-a} & a \leq u \leq b \\ 1 & u > b \end{cases}$$

`

Uniform Distribution

A few useful notes:

- The most commonly used uniform random variable is $U \sim Uniform(0, 1)$.
- Again, this is useful if we want to use a random variable that takes values within an interval, but we don't think it is likely to be in any certain region.
- The values a and b used to determine the range in which $f(u)$ is not 0 are parameters of the distribution.

Common Continuous Distributions

Exponential Distribution

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Exponential Distribution

Definition: Exponential random variable

An $\mathrm{Exp}(\alpha)$ random variable measures the waiting time until a specific event that has an equal chance of happening at any point in time. (it can be cosidered the continous version of geometric distribution)

> Examples:
>
> - Time between your arrival at the bus station and the moment that bus arrives
> - Time until the next person walks inside the park's library
> - The time (in hours) until a light bulb burns out.

Exponential Distribution

Definition: Exponential pdf
If X is an exponential random variable with rate $\frac{1}{\alpha}$ then the probability density function of X is given by

$$f(u) = \begin{cases} \dfrac{1}{\alpha} e^{-\frac{x}{\alpha}} & x \geq 0 \\ 0 & o.w. \end{cases}$$

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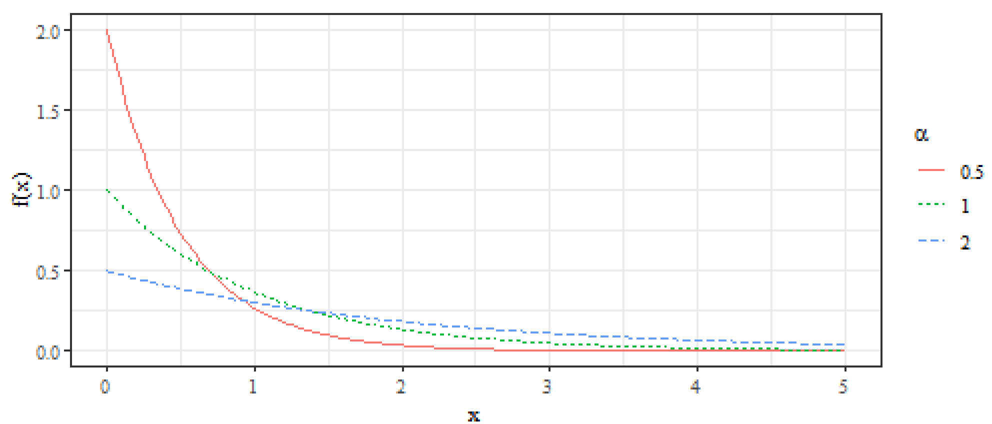
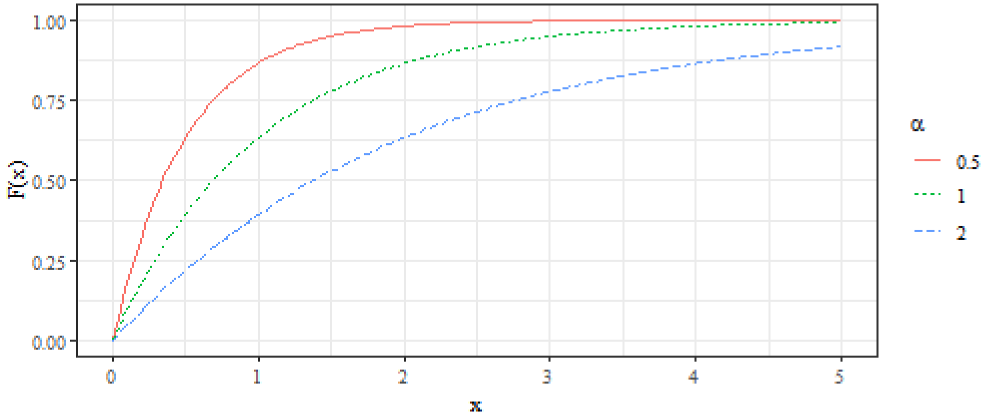


Exponential Distribution

Definition: Exponential CDF
If X is a exponential random variable with rate $1/\alpha$ then the cumulative density function of X is given by

$$F(x) = \begin{cases} 1 - exp(-x/\alpha) & 0 \leq x \\ 0 & x < 0 \end{cases}$$

`



Mean and Variance of Exponential Distribution

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Exponential Distribution

Definition: Exponential pdf
If X is an exponential random variable with rate $\frac{1}{\alpha}$ then the probability density function of X is given by

$$f(u) = \begin{cases} \dfrac{1}{\alpha} e^{-\frac{x}{\alpha}} & x \geq 0 \\ 0 & o.w. \end{cases}$$

` From this, we can derive:

$$E(X) = \alpha$$

$$Var(X) = \alpha^2$$

Exponential Distribution

Example: Library arrivals, cont'd

Recall the example the arrival rate of students at Parks library between 12:00 and 12:10pm early in the week to be about 12.5 students per minute. That translates to a $1/12.5 = .08$ minute average waiting time between student arrivals.

Consider observing the entrance to Parks library at exactly noon next Tuesday and define the random variable

$$T : \text{the waiting time (min) until the first} \\ \text{student passes through the door.}$$

Using $T \sim \text{Exp}(.08)$, what is the probability of waiting more than 10 seconds (1/6 min) for the first arrival?

Exponential Distribution

Example: Library arrivals, cont'd

$$T : \text{the waiting time (min) until the first student passes through the door.}$$

What is the probability of waiting less than 5 seconds?

Common Continous Distibutions

Normal Distribution

The Normal distribution

We have already seen the normal distribution as a "bell shaped" distribution, but we can formalize this.

The **normal** or **Gaussian** (μ, σ^2) distribution is a continuous probability distribution with probability density function (pdf)

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-\mu)^2/2\sigma^2} \qquad \text{for all } x$$

for $\sigma > 0$.

We then show that by $X \sim \mathrm{N}(\mu, \sigma^2)$

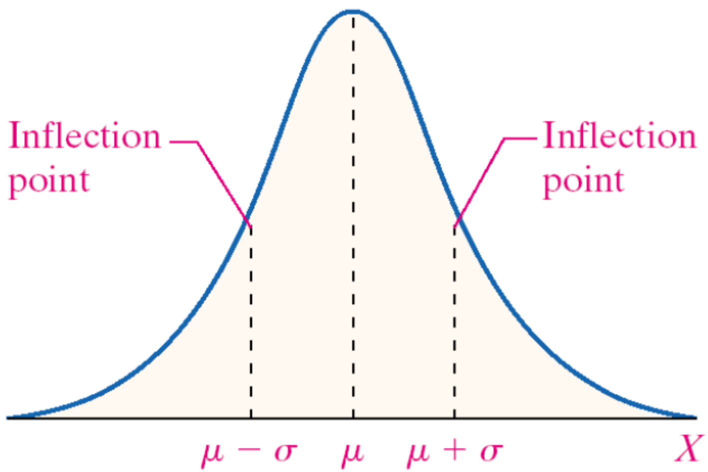
The Normal distribution

A normal random variable is (often) a finite average of many repeated, independent, identical trials.

> Mean width of the next 50 hexamine pallets
>
> Mean height of 30 students
>
> Total % yield of the next 10 runs of a chemical process

Normal Distribution's Center and Shape

Regardless of the values of μ and σ^2, the normal pdf has the following shape:



In other words, the distribution is centered around μ and has an inflection point at $\sigma = \sqrt{\sigma^2}$.

In this way, the value of μ determines the center of our distribution and the value of σ^2 deterimes the spread.

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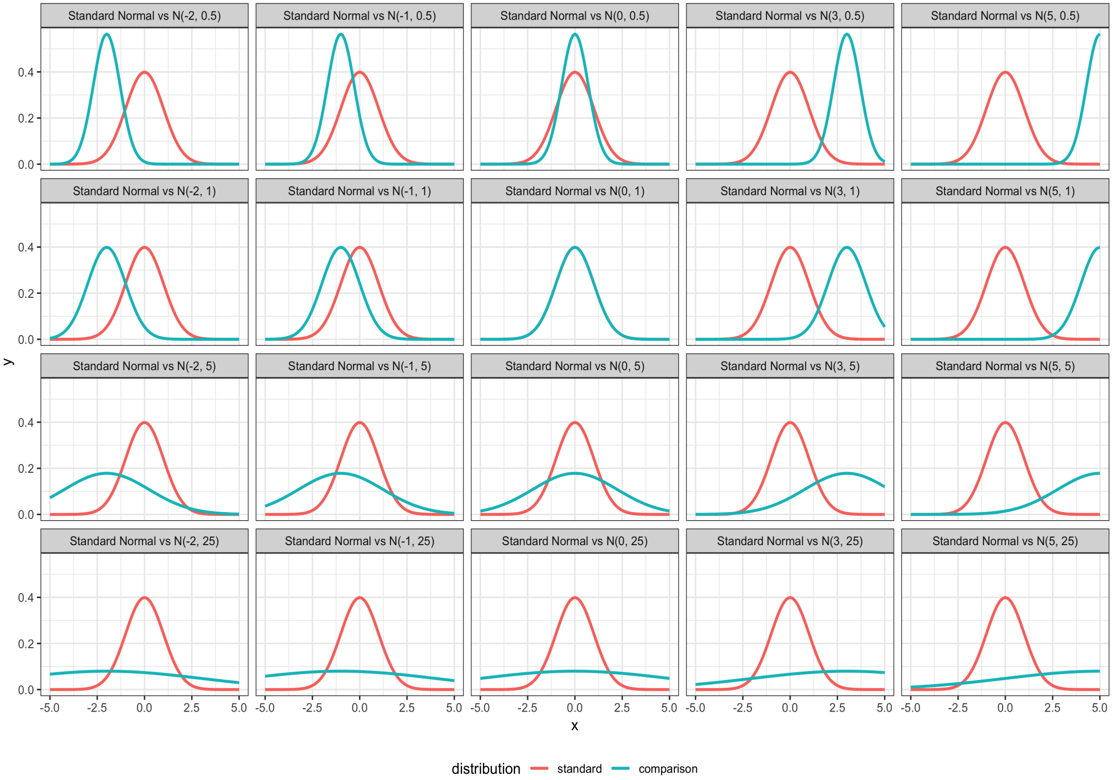
Normal Distribution's Center and Shape

Here we can see what differences in μ and σ^2 do to the shape of the shape of distribution



Mean and Variance

of

Normal Distribution

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The Normal distribution

It is not obvious, but

- $\int\limits_{-\infty}^{\infty} f(x)dx = \int\limits_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-\mu)^2/2\sigma^2} dx =$
- $\mathrm{E}X = \int\limits_{-\infty}^{\infty} x \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-\mu)^2/2\sigma^2} dx =$
- $\mathrm{Var}X = \int\limits_{-\infty}^{\infty} (x-\mu)^2 \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-\mu)^2/2\sigma^2} dx =$

One poine before we go on

Standardization

Definition

Standardization is the process of transforming a random variable, X, into the signed number of standard deviations by which it is is above its mean value.

$$Z = \frac{X - \mathrm{E}X}{\mathrm{SD}(X)}$$

Z has mean 0

Z has variance (and standard deviation) 1

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The Calculus I methods of evaluating integrals via anti-differentiation will fail when it comes to normal densities. They do not have anti-derivatives that are expressible in terms of elementary functions.

> This means we cannot find probabilities of a Normally distributed random variable by hand.
>
> So, what is the solution?
>
> Use computers or tables of values.

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The use of tables for evaluating normal probabilities depends on the following relationship. If $X \sim \text{Normal}(\mu, \sigma^2)$,

$$P[a \le X \le b] = \int_a^b \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-\mu)^2/2\sigma^2} dx$$

$$= \int_{(a-\mu)/\sigma}^{(b-\mu)/\sigma} \frac{1}{\sqrt{2\pi}} e^{-z^2/2} dz$$

$$= P\left[\frac{a-\mu}{\sigma} \le Z \le \frac{b-\mu}{\sigma}\right]$$

where $Z \sim \text{Normal}(0, 1)$.

Standard Normal Distribution

The parameters are important in determining the probability, but because the pdf of a normal random variable is difficult to work with we often use the distribution with $\mu = 0$ and $\sigma^2 = 1$ as a reference point.

> **Definition: Standard Normal Distribution**
> The standard normal distribution is a normal distribution with $\mu = 0$ and $\sigma^2 = 1$. It has pdf
>
> $$\begin{aligned} f(z) &= \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2} \\ &= \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}z^2\right) \end{aligned}$$
>
> `

We say that a random variable is a "standard normal random variable" if it follows a standard normal distribution or that $Z \sim N(0, 1)$.

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Standard Normal Distribution (cont)

It's worth pointing out the reason why the standard normal distribution is important. There is no "closed form" for the cdf of a normal distribution.

In other words, since we can't finish this step:

$$F(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2}(t-\mu)^2} dt = ???$$

we have to estimate the value each time. However, we have already done this for *standard* normal random variables already in **Table B.3**

So if $Z \sim N(0,1)$ then $P(Z \leq 1.5) = F(1.5) = 0.9332$.

The good news is that we can connect any normal probabilities to the values we have for the standard normal probabilities.

Standard Normal Distribution (cont)

These facts drive the connection between different normal random variables:

> **Key Facts: Converting Normal Distributions**
>
> If $X \sim N(\mu, \sigma^2)$ and $Z = \dfrac{X - \mu}{\sigma}$ then $Z \sim N(0, 1)$
>
> If $Z \sim N(0, 1)$ and $X = \sigma Z + \mu$ then $X \sim N(\mu, \sigma^2)$

We use this connection as a way to avoid working with the normal pdf directly.

Standard Normal Distribution (cont)

A rule of thumb in dealing with questions about finding probabilities of Normally distributed probabilities of $N(\mu, \sigma^2)$:

> (1) Translate that question to standard Normal distribution. i.e. $Z \sim N(0, 1)$
>
> (2) Look it up in a table

CDF of Standard Normal Distribution

The standard Normal distribution $ Z\sim N(0,1)$ plays an important rule in finding probabilities associated with a Normal random variable. The **CDF** of a standard Normal distribution is

$$\Phi(z) = F(z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-t^2} dt = P(Z \leq z).$$

Therefore, we can find probabilities for all normal distributions by tabulating probabilities for only the standard normal distribution. We will use a table of the **standard normal cumulative probability function**.

Standard Normal Distribution (cont)

Example: Normal to Standard Normal

If $X \sim N(3, 4)$ then:

$$P(X \leq 6) = P\left(\frac{X-3}{2} \leq \frac{6-3}{2}\right)$$

$$= P(Z \leq 1.5)$$

$$= 0.9332$$

where the valeu 0.9332 if found from **Table B.3**

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Standard Normal Distribution (cont)

Example: Standard normal probabilities

$P[Z < 1.76]$

$P[.57 < Z < 1.32]$

Table entry for z is the area under the standard normal curve to the left of z.

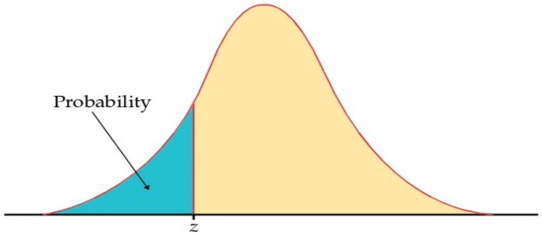

TABLE A

Standard normal probabilities

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

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- Exponential
- Normal
- **Std. Normal**

Table entry for z is the area under the standard normal curve to the left of z.

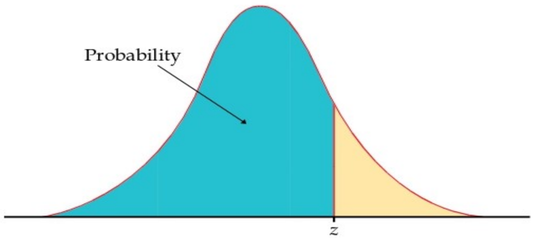

TABLE A

Standard normal probabilities (continued)

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

Some useful tips about standard Normal distribution

> By symmetry of the standard Normal distribution around zero $P(Z \geq a) = P(Z \leq -a)$

> We can also do it reverse, find z such that $P(-z \leq Z \leq z) = 0.95$
>
> $P(Z \geq \#) = 0.025$

Background

Terms and Use

Common Dists

Uniform

Exponential

Normal

Std. Normal

Example: Baby food

J. Fisher, in his article Computer Assisted Net Weight Control (**Quality Progress**, June 1983), discusses the filling of food containers with strained plums and tapioca by weight. The mean of the values portrayed is about 137.2g, the standard deviation is about 1.6g, and data look bell-shaped. Let

$$W = \text{the next fill weight.}$$

Let $W \sim N(137.2, 1.6^2)$. Find the probability that the next jar contains less food by mass than it's supposed to (declared weight = 135.05g).

More example

Using the standard normal table, calculate the following:

$P(X > 7), X \sim \text{Normal}(6, 9)$

$P(|X - 1| > 0.5), X \sim \text{Normal}(2, 4)$

More example

Find c such that

$$P(|X-2|>c)=0.01$$

where $X \sim \mathrm{Normal}(2,4)$