Quiz IV

STAT 305, Section 3
FALL 2019

Instructions

- The quiz is scheduled for 80 minutes, from 09:30 to 10:50 AM. At 10:50 AM the exam will end.
- Total points for the exam is 60. Points for individual questions are given at the beginning of each problem. Show all your calculations clearly to get full credit. Put final answers in the box at the right (except for the diagrams!).
- A forumula sheet is attached to the end of the exam. Feel free to tear it off.
- Normal quantile table is attached to the end of the exam. Feel free to tear it off.
- You may use a calculator during this exam.
- Answer the questions in the space provided. If you run out of room, continue on the back of the page.
- If you have any questions about, or need clarification on the meaning of an item on this exam, please ask your instructor. No other form of external help is permitted attempting to receive help or provide help to others will be considered cheating.
- **Do not cheat on this exam.** Academic integrity demands an honest and fair testing environment. Cheating will not be tolerated and will result in an immediate score of 0 on the exam and an incident report will be submitted to the dean's office.

Name: Key

Student ID:

1. In an effort to understand smartphone use, the American Association of Psychologists gained access to a single day's data on 625 smartphone users. They found that the smartphone users sampled spent an average of 270 minutes using their phone on that day with a standard deviation of phone use was 32.15 minutes.

The following table may be useful:

Table 1: z's for use in Two-sided Large-n intervals for the mean

Desired Confidence	z
80%	1.28
90%	1.645
95%	1.96
98%	2.33
99%	2.58

$\bar{x} = 270$

$n = 625 \ (> 25)$

$s = 32.15$

$1 - \frac{\alpha}{2} =$

(a) (4 points) Provide a two-sided 95% confidence interval for true number of minutes smartphone users spend using thier phones on average.

$z_{1-\alpha/2} = z_{.975} = 1.96$

Large "n" confidence interval:

$\alpha = 0.05$

$$\left(\bar{x} - z_{1-\alpha/2}\, s/\sqrt{n},\ \bar{x} + z_{1-\alpha/2}\, s/\sqrt{n}\right)$$

$$\left(270 - 1.96 \cdot \frac{32.15}{\sqrt{625}},\ 270 + 1.96\, \frac{32.15}{\sqrt{625}}\right) = (267.4794, 272.5206)$$

(b) (4 points) Provide a one-sided lower bound 99% confidence lower bound for the number of minutes smartphone users spend using thier phones on average.

$$\text{lower bound}: \left(\bar{x} - z_{1-\alpha}\, s/\sqrt{n},\ +\infty\right)$$

$$= \left(270 - 2.33 \cdot \frac{32.15}{\sqrt{625}},\ +\infty\right) = (267.0036,\ +\infty)$$

$z_{1-\alpha} = z_{0.99} \simeq 2.33$ (by Normal table)

(c) (10 points) A similar study two years ago determined that smartphone users spend an average of 4 hours per day using their phones. Perform a hypothesis test at the $\alpha = 0.05$ significance level for the claim that this is no longer the case.
Note: Write down all six steps for full credit.

4 hours = 240 minutes, so we want to test whether the actual average time is 240 or not

1, $H_0: \mu = 240$ vs. $H_a: \mu \neq 240$

2, $\alpha = 0.05$

3, $n \geq 25$, an σ is unknow, so we'll use the test statistic

$$K = \frac{\bar{X} - \mu_0}{S/\sqrt{n}} \quad \text{and} \quad K \sim N(0,1)$$

under H_0

4, $$K = \frac{\bar{x} - \mu_0}{S/\sqrt{n}} = \frac{270 - 240}{32.15/\sqrt{625}} = \frac{30}{\frac{32.15}{25}}$$

$$= 23.328$$

p-value = P(observing as or more extreme values than K = 23.328)

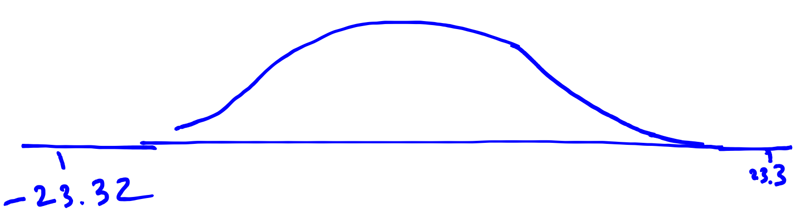
$$= P(|Z| > 23.328)$$

$$= P(Z > 23.328 \text{ or } Z < -23.328)$$

$$= \underbrace{P(Z > 23.328)}_{\simeq 0} + \underbrace{P(Z < -23.328)}_{\simeq 0}$$

(Note, we know that if $Z \sim N(0,1)$, $P(Z < -3) \simeq 0$ and $P(Z > +3) \simeq 0$)

$$= 0$$



5, Since the P-value (≈ 0) < 0.05, we reject the null Hypothesis.

6, There is enough evidence to conclude that the average time spent on smartphones is not 270 minutes.

2. A group of scientists are trying to understand the effects of temperature on two O-ring designs for a rocket. By placing the O-ring (attached to a valve) in a chamber and slowly lowering the chamber's temperature, the scientists are able to record the temperature at which the O-ring fails by monitoring when the valve begins to leak. After testing 10 O-rings for each type, the scientists find the mean failure temperature for the first O-ring design sample to be 50 K with a sample variance of 10 and the mean failure temperature of the second O-ring sample to be 53 K with a sample variance of 20.

(a) (4 points) Provide a one-sided lower bound 95% confidence interval for the true failure temperature of the **first** O-ring design.

$\alpha = 0.05$, $\bar{x} = 50$, $s_x^2 = 10$, $n_x = 10$

small sample confidence interval:

$\left(\bar{x} - t_{n-1, 1-\alpha}\sqrt{s_x^2/n_x}, +\infty\right)$

$= \left(50 - t_{9, 0.95}\frac{\sqrt{10}}{\sqrt{10}}, +\infty\right)$

$= (50 - 1.833, +\infty) = (48.167, +\infty)$

(b) (4 points) Provide a two-sided 95% confidence interval for the true failure temperature of the **second** O-ring design.

$\alpha = 0.05$, $\bar{y} = 53$, $s_y^2 = 20$, $n_y = 10$

$\bar{y} \pm t_{n-1, 1-\alpha/2}\sqrt{s_y^2/n_y} = 53 \pm t_{9, .975}\sqrt{\frac{20}{10}}$

$= 53 \pm 2.262\sqrt{2}$

$= (49.80105, 56.19895)$

(c) (6 points) Assume that the failure temperatures for both O-ring designs is normally distributed. Provide a one-sided upper bound 95% confidence interval for **difference in the two designs**.

small sample size CI for the difference in the two means:

$s_p^2 = \frac{(n_x-1)s_x^2 + (n_y-1)s_y^2}{n_x+n_y-2} = \frac{(9)(10)+(9)(20)}{18} = 15$

$= \left(-\infty, (\bar{x}-\bar{y}) + t_{n_x+n_y-2, 1-\alpha}\cdot\sqrt{\frac{s_p^2}{n_1}+\frac{s_p^2}{n_2}}\right)$

$= \left(-\infty, (50-53) + t_{18, .95}\sqrt{15/10 + 15/10}\right) = (-\infty, -3 + 1.734\cdot\sqrt{3})$

$(-\infty, 0.00337)$

(d) (10 points) Conduct a hypothesis test at the $\alpha = 0.05$ significance level for the claim that the true difference between the population means of the two O-rings ($\mu_X - \mu_Y$) are significantly different. Note: Write down all six steps for full credit.

1, $H_0: \mu_x - \mu_y = 0$ vs. $H_a: \mu_x - \mu_y \neq 0$

2, $\alpha = 0.05$

3, If we assume that ① H_0 is true ② sample 1 is iid $N(\mu_x, \sigma_x^2)$ ③ independent from sample 2 which is ④ iid $N(\mu_y, \sigma_y^2)$ and

(5) $\sigma_x^2 \approx \sigma_y^2$, we can then use the test statistic

$$K = \frac{\bar{x} - \bar{y} - 0}{\sqrt{\frac{S_p^2}{n_x} + \frac{S_p^2}{n_y}}} \sim t_{(n_x+n_y-2)}$$

4/ $$K = \frac{50-53}{\sqrt{\frac{15}{10}+\frac{15}{10}}} = \frac{-3}{\sqrt{3}} = -1.732$$

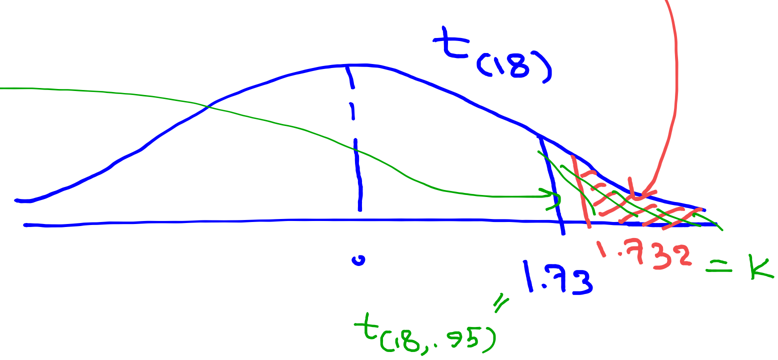
corresponding to α: $t_{(n_x+n_y-2,\,1-\alpha)} = t_{(18,\,.95)}$

by t-table $= 1.73$

Note: by the methods I discussed in the slides, we just need to determine whether p-value $\gtrless \alpha$.

corresponding to p-value, $K = |-1.732|$

So, $\begin{cases} \text{p-value} = P(|T| > |K|) = P(|T| > 1.732) \\ t_{(18,\,0.95)} = 1.73 \end{cases}$



(The area under the curve for p-value is smaller

then the area corresponding to α $(t_{(18,\ .95)})$.

So, p-value $< \alpha$.

5, Since p-value $< \alpha$, we reject the null

hypotesis.

6, There is enough evidence to conclude that

on average the true failure temperature of the two O-rings

are not the same $(\mu_1 - \mu_2 \neq 0)$.

(we can also say that on average the true

failure temperature for o-ring type I is 3°

less than o-ring type II)

3. An arctic research station recently did a major overhaul to their server system hardware and the technicians are checking to make sure that there has been no loss in download speed. The previous download speed had an average of 63.4 Mbps. A systems analyst took 10 readings on the download speeds during the course of a day to check. Her results are below (in Mbps):

62.87, 63.04, 62.83, 63.32, 63.07, 62.84, 63.1, 63.15, 63.12, 62.94

The sample average is $\bar{x}$ 63.03 and the sample variance is 0.025. s^2

(a) (4 points) Provide a two-sided 95% confidence interval for the mean download speed.

Small sample: CI

$$\bar{x} \pm t_{n-1, 1-\alpha/2} \sqrt{\frac{s^2}{n}} \Rightarrow 63.03 \pm t_{9, .975} \cdot \sqrt{\frac{0.025}{10}}$$

$t_{9, .975} = 2.262$

$$\Rightarrow 63.03 \pm 2.262\,(0.05)$$

$$= (62.9169,\ 63.1431)$$

(b) (4 points) Provide a one sided 95% lower confidence bound for the mean download speed.

$$\left(\bar{x} - t_{n-1, 1-\alpha} \sqrt{s^2/n},\ +\infty\right)$$

$t_{9, 0.95} = 1.833$

$$\left(63.03 - 1.833 \sqrt{\frac{0.025}{10}},\ +\infty\right) = (62.93835,\ +\infty)$$

(c) (10 points) Conduct a hypothesis test at the 95% confidence level for the null hypothesis $\mu \geq 63.4$ against the alternative $\mu < 63.4$. Include your hypothesis statement, the choice of test statistic, the p-value, and your conclusion.
Note: Write down all six steps for full credit.

1, $H_0: \mu \geq \underbrace{63.4}_{\mu_0}$ vs. $H_a: \mu < 63.4$

2, $\alpha = 0.05$

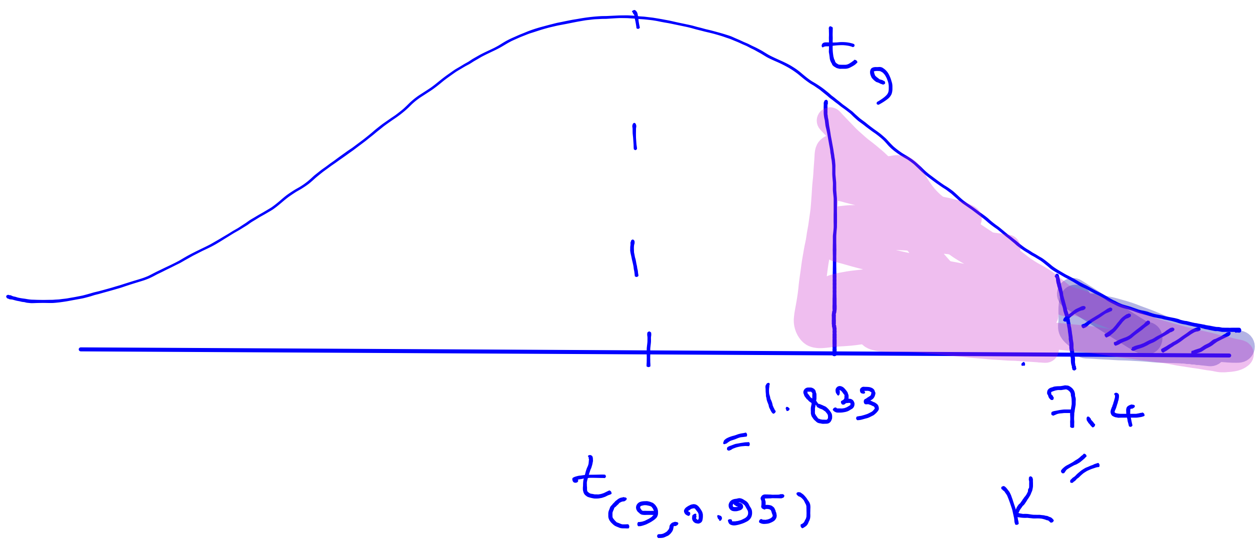
3, Since $n < 25$ and σ^2 is Unknown, I will use the test statistic if H_0 is true and $\bar{x} \sim N(\mu, \sigma^2/n)$:

$$K = \frac{\bar{x} - \mu_0}{\sqrt{s^2/n}} \sim t_{(n-1)}$$

4, $k = \dfrac{63.03 - 63.4}{\sqrt{\dfrac{0.025}{10}}} = \dfrac{-0.37}{0.05} = -7.4$

corresponding to α: $t_{(9, 0.95)} = 1.833$

corresponding to p-value: $|k| = |-7.4|$



area under the curve corresponding to p-value $<$ area under the curve corresponding to α.

So, p-value $< \alpha$ & we reject the null.

6, there is enough evidence that the average download speed is not ≥ 63.4.