1. Let X be a random variable with support $=\{0,1,2,3\}$.

① $P(x) \geq 0 \ \forall x \in S_x$

② $\sum_{x \in S_x} P(x) = 1$

(a) Fill in the blank in the table below to make it a valid probability mass function:

x	0	1	2	3
$P_X(x)$	0.5	0.25	0.1	? ≥ 0

$$\sum_{x \in S_x} P(x) = 1 \Rightarrow 0.5 + 0.25 + 0.1 + ? = 1$$

$$\Rightarrow ? = 0.15$$

(b) Derive the cumulative distribution function for X.

$F(0) = P(X \leq 0) = P(X=0) = 0.5$

$F(1) = P(X \leq 1) = P(X=0) + P(X=1) = 0.5 + 0.25 = 0.75$

$F(2) = P(X \leq 2) = P(X=0) + P(X=1) + P(X=2) = 0.5 + 0.25 + 0.1 = 0.85$

X	0	1	2	3
P(x)	0.5	0.25	0.1	0.15
F(x)	0.5	0.75	0.85	1

$F(3) = P(X \leq 3) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$

$= 0.5 + 0.25 + 0.1 + 0.15 = 1$

Determine the probabilities that:

(c) X is at least 2.

$$P(X \geq 2) = 1 - P(X<2)$$

$$= 1 - \boxed{P(X \leq 1)} = 1 - \boxed{F(1)} = 1 - 0.75 = 0.25$$

(d) X is neither 0 nor 2.

$$P(X \notin \{0,2\}) = P(X=1 \text{ or } X=3)$$

$$= P(X=1) + P(X=3) = 0.25 + 0.15 = 0.4$$

(e) X is non-negative.

$$P(X \geq 0) = 1 \qquad P(X \in \mathbb{N}) = 1 - P(X=0)$$

$$= 0.5$$

(f) Find the expected value of X.

$$E(X) = \sum_{x \in \{0,1,2,3\}} x \cdot P(x) = 0 \cdot P(X=0) + 1 \cdot P(X=1) + 2\,P(X=2) + 3 \cdot P(X=3)$$

$$= 0(0.5) + 1 \cdot (0.25) + 2(0.1) + 3(0.15)$$

$$= 0 + 0.25 + 0.2 + 0.45 = 0.9$$

(g) Find the variance of X.

$$Var(X) = \sum_{x \in \{0,1,2,3\}} (x_i - E(X))^2 P(x) = (0-0.9)^2 P(X=0) + (1-0.9)^2 P(X=1)$$

$$+ (2-0.9)^2 P(X=2) + (3-0.9) P(X=3)$$

$$= 1.19$$

2. Let X be a random variable with the following distribution with probability function

$$f(x) = \begin{cases} \frac{c}{x} & x = 1, 2, 3, 4 \\ \\ 0 & o.w. \end{cases}$$

where c is a constant.

(a) Find the value of c that makes $f(x)$ a valid probability function.

$\sum_{x=1}^{4} P(X=x) = 1 \implies \sum_{x=1}^{4} c/x = 1 \implies c \sum_{x=1}^{4} 1/x = 1 \implies c(1 + 1/2 + 1/3 + 1/4) = 1$

$\implies c(2.08) = 1$

$\implies c = \frac{1}{2.08} = \boxed{0.48}$

(b) Find the value of $E(X)$.

$EX = \sum_{x=1}^{4} x \cdot f(x) = \left(1\left(\frac{0.48}{1}\right) + 2\left(\frac{0.48}{2}\right) + 3\left(\frac{0.48}{3}\right) + 4\left(\frac{0.48}{4}\right)\right)$

$= 4(0.48) = 1.92$

(c) Find the value of σ^2 for this random variable.

$\sigma^2 = \text{Var } X = EX^2 - (EX)^2$

$EX^2 = \sum_{x=1}^{4} x^2 f(x) = \left(1^2\frac{(0.48)}{1} + 2^2\frac{(0.48)}{2} + 3^2\frac{(0.48)}{3} + 4^2\frac{(0.48)}{4}\right) = 10(0.48) = \boxed{4.8}$

3. Let X be a random variable following a binomial distribution with probability function

$X \sim \text{binomial}(4, 0.6)$ $\quad f(x) = \frac{4!}{x!(4-x)!}(0.6)^x(0.4)^{4-x}$

. Complete the probability table for X and find the mean and CDF of X.

x	$P(X = x)$	$F_X(x)$	$x\,F_X(x)$
0	$\frac{4!}{0!(4-0)!}(0.6)^0(0.4)^4 = 0.0256$	0.0256	$0(0.0256) = 0$
1	0.1536	0.1792	$1(0.1536) = 0.1536$
2	0.3456	0.5248	$2(0.3456) = 0.6912$
3	0.3456	0.8704	$3(0.3456) = 1.0368$
4	0.1296	1	$4(0.1296) = 0.5184$

$EX = \sum_{x=0}^{4} x f(x) \approx 2.4$

In binomial distribution, $EX = n.p$
$= 4(0.6) = \boxed{2.4}$