1. A lab network consisting of 20 computers was attacked by a computer virus. This virus enters each computer with probability $p = 0.4$ independently of other computers.

   (a) Is the random variable associated with this random experiment **discrete** or **continuous**?

   (b) **Precisely** specify the probability distribution that best matches with this random experiment.

   \# of virus enters : $X \sim \text{binom}(n=20, p=0.4)$, $f_X(x) = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$

   Find the probability that

   (c) the virus entered exactly none or only one computer.

   $P(\text{"exactly none or exactly one enters"}) = P(X=0 \text{ OR } X=1) \overset{\text{indep}}{=} P(X=0) + P(X=1) = \frac{20!}{0!(20-0)!}(0.4)^0(1-0.4)^{20} + \frac{20!}{1!(20-1)!}(0.4)^1(1-0.4)^{20-1} = 0.00052$

   (d) the virus entered at least two computers.

   $P(\text{"at least two enters"}) = P(X \geq 2)$

   $P(X=2 \text{ or } X=3 \ldots X=20)$

   $P(X \geq 2) = 1 - P(X < 2) = 1 - P(X \leq 1) = 1 - [P(X=0 \text{ OR } X=1)]$

   $= 1 - 0.00052 = 0.9994$

   (e) What is the standard deviation of the number of times the virus enters?

   $X \sim \text{binomial}(20, 0.4) \Longrightarrow EX = n.p = 20(0.4) = 8$, $Var(X) = n.p(1-p) = 20(0.4)(0.6) = 4.8$

   $SD(X) = \sqrt{Var X} = \sqrt{4.8} = 2.19$

2. Suppose that the average number of log-ins to a Statistics department server is 6 per minute. Let X be the random variable describing the average number of log-ins.

   (a) What are possible values of X? Does it take **discrete** or **continous** values?

   X: Average # of log-ins : $0, 1, 2, \ldots$

   (b) **Precisely** specify the probability distribution that best matches with the average number of log-ins. What is the probability of

   $X \sim \text{poisson}(\lambda = 6) \to$ 6 log-ins PER minute. , $f_X(x) = \frac{e^{-\lambda}\lambda^x}{x!}$

   (c) No log-ins in a 1-minute period?

   $P(X=0) = \frac{e^{-6}6^0}{0!} = e^{-6} = 0.0024$

   (d) Maximum log-ins of two in a 1-minute period?

   $P(\text{"max log-ins of 2"}) = P(X \leq 2) = \sum_{x=0}^{2} \frac{e^{-6}6^x}{x!} = P(X=0) + P(X=1) + P(X=2)$

   $= \frac{e^{-6}6^0}{0!} + \frac{e^{-6}6^1}{1!} + \frac{e^{-6}6^2}{2!} = e^{-6}\underbrace{\left(1 + 6 + \frac{6^2}{2!}\right)}_{7+18} = e^{-6}(25) = 0.0619$

   (e) At least three log-ins in a 1-minute period?

   $P(X \geq 3) = \sum_{x=3}^{\infty} \frac{e^{-6}6^x}{x!} = \ldots$

   use the complement trick: $P(X \geq 3) = 1 - P(X < 3) = 1 - P(X \leq 2) \overset{\text{Part (d)}}{=} 1 - 0.0619$

   $= 0.9381$

$X \sim \text{Poisson}(\lambda=6) \Longrightarrow EX = \text{Var}\,X = \lambda = 6$

(f) What is the expected value and variance of the average number of log-ins?

3. A representative from the National Football League's Marketing Division randomly selects people on a random street in Kansas City, Kansas until he finds a person who attended the last home football game. Let p, the probability that he succeeds in finding such a person, equal 0.20. And, let X denote the number of people he selects until he finds his first success.

   (a) What is the distribution of X?

   $X \sim \text{Geom}(p=0.2)$, $f(x) = p(1-p)^{x-1}$, $x = 1, 2, 3, \ldots$

   (b) What is the probability that the marketing representative must select 4 people before he finds one who attended the last home football game?

   $P(X=4) = p(1-p)^{4-1} = (0.2)(1-0.2)^3$
   $= (0.2)(0.8)^3 = 0.1024$

   (c) What is the probability that he finds someone who attended the last home football game after he approaches 21 people?

   P("He approaches 21 people & still no successes")
   = P("He should approach more than 21 people for the first success")
   $= P(X \geq 21) = 1 - P(X < 21) = 1 - P(X \leq 20) = 1 - F_X(21) = 1 - [1-(1-0.2)^{20}]$
   $= (0.8)^{20} = 0.0115$

4. An experiment was conducted to examine aluminum alloy for surface flaws. The average number of sheets with a given number of flaws per sheet was 3.

   (a) **Precisely** specify the probability distribution that best describes this experiment.

   $X :=$ average number of flaws per sheet $\sim \text{Pois}(\lambda=3)$

   (b) What is the probability of finding a sheet chosen at random which contains 3 or more surface flaws?

   $P(\text{"3 or more flaws"}) = P(X \geq 3) = \sum_{x=3}^{\infty} \frac{e^{-3} 3^x}{x!} = \ldots$

   $1 - P(X<3) = 1 - P(X \leq 2)$
   $= 1 - [P(X=0) + P(X=1) + P(X=2)] = \ldots$
   $= 0.7759$

   (c) Find the expected value and variance of flaws.

5. A safety engineer feels that 35% of all industrial accidents in her plant are caused by failure of employees to follow instructions. She decides to look at the accident reports (selected randomly and replaced in the pile after reading) until she finds one that shows an accident caused by failure of employees to follow instructions.

   $X \sim \text{Geom}(p=0.35)$

   Expected value of X, $X \sim \text{Geom}(p=0.35) \Rightarrow EX = \frac{1}{p} = \frac{1}{0.35} = 28.75$

   $f_X(x) = p(1-p)^{x-1}$

   $F_X(x) = 1-(1-p)^x$

   CDF

   (a) On average, how many reports would the safety engineer expect to look at until she finds a report showing an accident caused by employee failure to follow instructions?

   (b) Find $P(X \geq 3)$ $= 1 - P(X<3) = 1 - P(X \leq 2) = 1 - F_X(2) = 1 - [1-(1-p)^x]$
   $= (1-0.35)^2 = (0.65)^2 = 0.4225$

Use the Compliment trick