1. Find the expected value and variance of a continuous distribution with follwing probability density:

$$f(x) = \begin{cases} 0.3 & 0 < x < 1 \\ 0.7 & 1 < x < 2 \\ 0 & o.w \end{cases}$$

$Ex = \int_{S_x} x f(x) dx = \int_0^2 x f(x) dx$

$= \int_0^1 x f(x) dx + \int_1^2 x f(x) dx = \int_0^1 x(0.3) dx + \int_1^2 x(0.7) dx$

$= (0.3)\int_0^1 x dx + 0.7 \int_1^2 x dx = 0.3 \cdot \frac{x^2}{2}\Big|_0^1 + 0.7 \frac{x^2}{2}\Big|_1^2 = 0.3(1^2/2 - 0) + 0.7(2^2/2 - 1^2/2)$

$Ex^2 = \int_{S_x} x^2 f(x) dx = \int_0^1 x^2 f(x) dx + \int_1^2 x^2 f(x) dx = \int_0^1 x^2 (0.3) dx + \int_1^2 x^2 (0.7) dx$ $= 0.3(1/2) + 0.7(3/2) = 1.2$

$= 0.3 \frac{x^3}{3}\Big|_0^1 + 0.7 \frac{x^3}{3}\Big|_1^2 = 0.3(1^3/3 - 0) + 0.7(2^3/3 - 1^3/3)$

$= 0.3(1/3) + 0.7(7/8) = 1.73$

$\Rightarrow Var X = Ex^2 - (Ex)^2$

$= 1.73 - (1.2)^2$

$= 0.29$

2. X has a continuous distribution with the following *pdf*.

$$f(x) = \begin{cases} c\exp(-2x) & 0 < x < \infty \\ 0 & o.w \end{cases}$$

(a) Find c such that $f(x)$ is a pdf. (need to find c such that $\int_0^\infty f(x) dx = 1$

$1 = \int_0^\infty f(x) dx = \int_0^\infty c e^{-2x} dx = c\int_0^\infty e^{-2x} dx = c\left(-\frac{1}{2} e^{-2x}\Big|_0^\infty\right) = c\left(-\frac{1}{2}\left[e^{-2(\infty)} - e^{-2(0)}\right]\right)$

$\Rightarrow \quad 1 = c(-1/2(0-1)) \Rightarrow 1 = c(1/2) \Rightarrow \boxed{c = 2}$

(b) Find $P(X \geq 5)$

$P(X \geq 5) = \int_5^\infty 2e^{-2x} dx = -e^{-2x}\Big|_5^\infty = -\left[\underbrace{e^{-2(\infty)}}_{0} - \underbrace{e^{-2(5)}}_{e^{-10}}\right]$

$= -(0 - e^{-10}) = e^{-10} \simeq 4.53 \times 10^{-5}$

(c) Find the $E(X)$

$E X = \int_0^\infty x f(x) dx = \int_0^\infty \underbrace{x}_{u} \underbrace{2e^{-2x} dx}_{dv}$

Note: Integral by parts:

$\int u dv = uv - \int v du$

$\Rightarrow \begin{cases} u = x \longrightarrow \boxed{du = dx} \\ \int dv = \int 2e^{-2x} dx \Rightarrow v = -e^{-2x} \end{cases}$

$\longrightarrow \int_0^\infty x 2e^{-2x} dx = x(-e^{-2x})\Big|_0^\infty - \int_0^\infty -e^{-2x} dx$

$$= -xe^{-2x}\Big|_0^\infty + \int_0^\infty e^{-2x}\,dx$$

$$= \underbrace{-(\infty)e^{-2(\infty)}}_{0} + \underbrace{0\,e^{-2(0)}}_{0} + \left(-\tfrac{1}{2}e^{-2x}\Big|_0^\infty\right)$$

$$= 0 - \tfrac{1}{2}e^{-2x}\Big|_0^\infty$$

$$= -\tfrac{1}{2}\Big(\underbrace{e^{-2(\infty)}}_{0} - \underbrace{e^{-2(0)}}_{1}\Big) = -\tfrac{1}{2}(0-1)$$

$$\boxed{= \tfrac{1}{2}}$$

3. Suppose that X is a NOrmal random variable with mean $\mu = 10.2$ and standard deviation $\sigma = 0.7$. Evaluate the following probabilities:

(a) $P(X \le 10.1) = P\left(\frac{X-10.2}{0.7} \le \frac{10.1-10.2}{0.7} \right) = P\left(Z \le \frac{-0.1}{0.7}\right) = \Phi(-0.14) = 0.4432$

$Z \sim N(0,1)$

(b) $P(9.0 < X \le 10.3) = P\left(\frac{9-\mu}{\sigma} < \frac{X-\mu}{\sigma} < \frac{10.3-\mu}{\sigma} \right) = P\left(\frac{9-10.2}{0.7} < Z < \frac{10.3-10.2}{0.7} \right)$

$= P(-1.71 < Z < 0.14) = P(Z < 0.14) - P(Z < -1.71)$

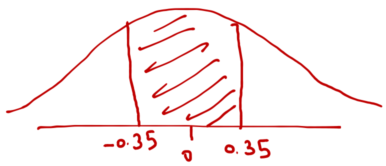
$= \Phi(0.14) - \Phi(-1.71) = 0.5556 - 0.0436$

$= 0.512$

(c) $P(|X - \overset{\mu}{10.2}| \le 0.25)$

$= P(-0.25 \le X - 10.2 \le 0.25)$

$= P\left(\frac{-0.25}{0.7} \le \frac{X-10.2}{0.7} \le \frac{0.25}{0.7} \right)$



$= P(-0.35 \le Z \le 0.35) =$ (1) $P(Z \le 0.35) - P(Z \le -0.35) = \Phi(0.35) - \Phi(-.35) = 0.6368 - 0.3631$

$= 0.2737$

using the symmetry of Normal $=$ (2) $1 - 2P(Z \le -0.35) = 1 - 2\Phi(-0.35) = 1 - 2(0.3631)$

$= 1 - 0.7262$

$= 0.2737$

Find numbers # such that the following statement s about X are true:

(a) $P(|X - 10.2| \ge \#) = 0.8$

$0.8 = P(|X - 10.2| \ge \#) = P(X - 10.2 \ge \# \cup X - 10.2 \le -\#)$

$= P(X - 10.2 \ge \#) + P(X - 10.2 \le -\#)$

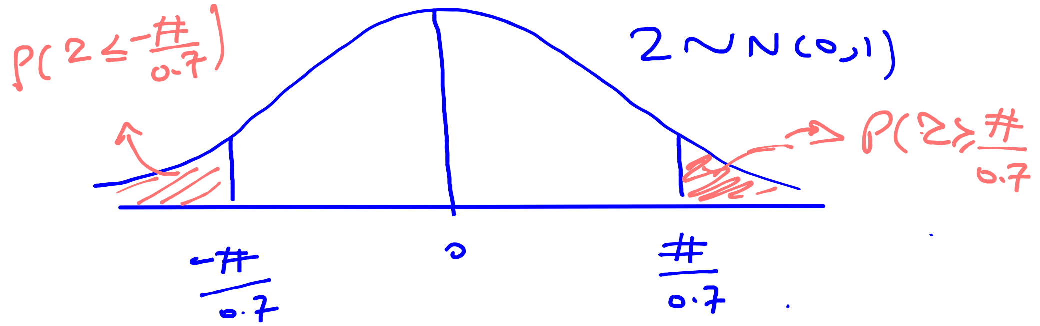
$= P(X \ge \# + 10.2) + P(X \le -\# + 10.2)$

$= P\left(\frac{X-10.2}{0.7} \ge \frac{\# \cancel{+10.2} \cancel{-10.2}}{0.7} \right) + P\left(\frac{X-10.2}{0.7} \le \frac{-\# \cancel{+10.2} \cancel{-10.2}}{0.7} \right)$

$= P\left(Z \ge \frac{\#}{0.7}\right) + P\left(Z \le \frac{-\#}{0.7}\right)$

(b) $P(X < \#) = 0.8$

$\boxed{P(Z \ge a) = P(Z \le -a)}$ $= 2P\left(Z \le \frac{-\#}{0.7}\right)$



$$\Rightarrow 0.4 = \frac{0.8}{2} = P\left(Z \le \frac{-\#}{0.7}\right) = \Phi\left(\frac{-\#}{0.7}\right)$$

$$-1.75 = \frac{-\#}{0.7}$$

$$\Rightarrow \# = 0.7(1.75)$$

Part (b)

$$P(X \le \#) = 0.8$$

$$\Rightarrow 0.8 = P\left(\frac{X - 10.2}{0.7} \le \frac{\# - 10.2}{0.7}\right)$$

$$= P\left(Z \le \frac{\# - 10.2}{0.7}\right)$$

$$= \Phi\left(\frac{\# - 10.2}{0.7}\right)$$

by the table $\Rightarrow \dfrac{\# - 10.2}{0.7} = 0.8416$

$$\boxed{\Rightarrow \# = 10.7891}$$