Example 6.7. Suppose you are a manufacturer of construction equipment. You make 0.0125 inch wire rope and need to determine how much weight it can hold before breaking so that you can label it clearly. Here are breaking strengths, in kg, for 41 sample wires:

```
 [1] 100.37  96.31  72.57  88.02 105.89 107.80  75.84  92.73  67.47  94.87
[11] 122.04 115.12  95.24 119.75 114.83 101.79  80.90  96.10 118.51 109.66
[21]  88.07  56.29  86.50  57.62  74.70  92.53  86.25  82.56  97.96  94.92
[31]  62.00  93.00  98.44 119.37 103.70  72.40  71.29 107.24  64.82  93.51
[41]  86.97
```

The sample mean breaking strength is ($\bar{x}=$) 91.85 kg and the sample standard deviation is ($s=$) 17.6 kg. Using the appropriate 95% confidence interval, try to determine whether the breaking strengths meet the requirement of at least 85 kg. => one-sided CI w/ lowerbound

$1-\alpha = .95 \Rightarrow \alpha = .05$

$\bar{x} = 91.85$

$s = 17.6$

$n = 41$

$$\left(\bar{x} - z_{1-\alpha}\frac{s}{\sqrt{n}},\ \infty\right)$$

$$= \left(91.85 - z_{.95}\frac{17.6}{\sqrt{41}},\ \infty\right)$$

$$= \left(91.85 - 1.64\left(\frac{17.6}{\sqrt{41}}\right),\ \infty\right)$$

$$= (87.3422,\ \infty)$$

With 95% confidence, we have shown that the true mean breaking strength is above 87.3422 kg. Hence, we meet the 85 kg requirement with 95% confidence.

Hence:

$$0.95 = P(Z > \frac{c-2}{\sqrt{0.154}})$$

$$0.95 = 1 - P(Z \leq \frac{c-2}{\sqrt{0.154}})$$

$$\begin{aligned}
0.05 &= P(Z \leq \frac{c-2}{\sqrt{0.154}}) \\
0.05 &= \Phi(\frac{c-2}{\sqrt{0.154}}) \\
\Phi^{-1}(0.05) &= \frac{c-2}{\sqrt{0.154}} \\
-1.64 &= \frac{c-2}{\sqrt{0.154}} \\
1.36 &= c
\end{aligned}$$

c. $P(|\overline{X} - 5| > 1)$, where $X_1, X_2, \ldots, X_{38} \sim$ are iid each with mean 5 and variance 10.

By the Central Limit Theorem, $\overline{X} \sim$ approx. $N(5, 10/38) = N(5, 0.263)$

$$\begin{aligned}
P(|\overline{X} - 5| > 1) &= P(\overline{X} - 5 > 1) + P(\overline{X} - 5 < -1) \\
&= P\left(\frac{\overline{X} - 5}{\sqrt{0.263}} > \frac{1}{\sqrt{0.263}}) + P(\frac{\overline{X} - 5}{\sqrt{0.263}} < \frac{-1}{\sqrt{0.263}}\right) \\
&\approx P(Z > 1.95) + P(Z < -1.95) \\
&= 2P(Z \leq -1.95) \\
&= 0.0512
\end{aligned}$$

Exercise 3.5.
David Bowie wanted to measure the quality of his music. So on n =123 separate occasions, he flew to Tibet to perform for the Dalai Lama. On each visit, after meditation, tea, and a contemplative friendly discussion, Bowie performed a new piece, and the wise Lama rated the morality of Bowie's music on a scale from 0 to 100 (100 = perfectly virtuous, 0 = nauseatingly

abominable). (Disclaimer: this question is complete fiction.)

The sample mean of the morality ratings was $\overline{x}$ =77.89, which is an estimate of the true mean morality rating μ of David Bowie's music. The sample standard deviation of the ratings was $s = 10$. Use the above information to answer the following:

a. In his benevolence and jest, the Dalai Lama proclaims a musician "enlightened" if his true mean morality rating is above 75. Is there enough evidence at significance level $\alpha = 0.1$ to suggest that David Bowie is enlightened?

1. **State the hypotheses and the significance value:**

$$
\begin{aligned}
H_0 &: \mu = 75 \\
H_a &: \mu > 75 \\
\alpha &= 0.1
\end{aligned}
$$

2. **Find the test statistic and state the distribution it came from:**

Since $n = 123 \geq 25$, the test statistic is:

$$K = \frac{\overline{x} - 75}{s/\sqrt{n}} = \frac{77.89 - 75}{10/\sqrt{123}} = 3.21$$

which came from:

$$K \sim \text{ approx. } N(0,1)$$

3. **Compute the p-value, or at least compare it to α:**

Since we have a one-sided upper alternative hypothesis and a test statistic from an approximate standard normal distribution, the p-value is:

$$P(Z > K) = P(Z > 3.21) = 1 - P(Z \leq 3.21) = 1 - 0.9993 = 0.0007$$

4. **Make a decision in terms of the hypotheses:**

   Since the p-value is $< \alpha$, we reject H_0 and conclude H_a.

5. **State a conclusion in the context of the problem:**

   We have enough evidence to claim that David Bowie's true mean morality rating is above 75: i.e., that David Bowie is enlightened.

b. Calculate and interpret a 2-sided 95% confidence interval for David Bowie's true mean morality rating.

Since $n = 123 \geq 40$, $1 - \alpha = 0.95$, and σ is unknown, our confidence interval is:

$$\left(\overline{x} - z_{1-\alpha/2}\frac{s}{\sqrt{n}},\ \overline{x} + z_{1-\alpha/2}\frac{s}{\sqrt{n}}\right)$$

$$\left(\overline{x} - z_{0.975}\frac{s}{\sqrt{n}},\ \overline{x} + z_{0.975}\frac{s}{\sqrt{n}}\right)$$

$$\left(77.89 - 1.96 \cdot \frac{10}{\sqrt{123}},\ 77.89 + 1.96\frac{10}{\sqrt{123}}\right)$$

$$(76.12, 79.66)$$

We are 95% confident that David Bowie's true morality rating μ lies in the interval, (76.12, 79.66).

c. In another inside jest, the Dalai Lama proclaims a musician to be in the "Sage Range" if his true mean morality rating is greater than 60 and less than 80. Based on the confidence interval in part (b), are you at least 95% confident that David Bowie is in the Sage Range?

We're 95% confident that David Bowie's true morality rating is inside (

76.12, 79.66), which in turn, is inside the Sage Range of (60, 80). Hence, we're at least 95% confident that David Bowie is in the Sage Range.

Exercise 3.6.
You are a manufacturer of the Fidget Widget©: a little device that, when placed on one's head, massages the subjects temples to curb fidgeting during class (again, fiction). You need to make sure that this device's true mean mass is close to 5 kg. Too high and the subject's neck will strain, but too low and the device is either flimsy or missing some components.

Suppose you take a sample of 41 newly-made Fidget Widgets and calculate their sample mean mass to be 6 kg. Suppose the standard deviation of the sample masses is 0.005 kg.

a. Is there enough evidence to conclude that the true mean mass of the Fidget Widgets is different from the target of 5 kg? Do a hypothesis test at significance level $\alpha = 0.01$ to justify your answer.

1. **State the hypotheses and the significance value:**

$$H_0 : \mu = 5$$
$$H_a : \mu \neq 5$$
$$\alpha = 0.01$$

2. **Find the test statistic and state the distribution it came from:**

Since $n = 41 \geq 25$, the test statistic is:

$$K = \frac{\bar{x} - 5}{s/\sqrt{n}} = \frac{6 - 5}{0.005/\sqrt{41}} = 1280.625$$

which came from:

$$K \sim \text{ approx. } N(0, 1)$$

3. **Compute the p-value, or at least compare it to α:**

   Since we have a one-sided upper alternative hypothesis and a test statistic from an approximate standard normal distribution, the p-value is:

   $P(|Z \ | > |K|) = P(Z \ > 1280.625) + P(Z \ < -1280.625) \approx 0 + 0 = 0$

4. **Make a decision in terms of the hypotheses:**

   Since the p-value is $\approx 0 < \alpha$, we reject H_0 and conclude H_a.

5. **State a conclusion in the context of the problem:**

   We have enough evidence to claim that the true mean weight of the Fidget Widgets produced is different from the target weight of 5 kg.

b. Calculate a two-sided 95% confidence interval for the true mean mass. Based on the confidence interval, is 5 kg a plausible guess for the true mean mass of the Fidget Widgets?

Since $n = 41 \geq 40$, $1 - \alpha = 0.95$, and σ is unknown, our confidence interval is:

$$\left(\overline{x} - z_{1-\alpha/2}\frac{s}{\sqrt{n}},\ \overline{x} + z_{1-\alpha/2}\frac{s}{\sqrt{n}}\right)$$

$$\left(\overline{x} - z_{0.975}\frac{s}{\sqrt{n}},\ \overline{x} + z_{0.975}\frac{s}{\sqrt{n}}\right)$$

$$\left(6 - 1.96 \cdot \frac{0.005}{\sqrt{41}},\ 6 + 1.96\frac{0.005}{\sqrt{41}}\right)$$

$$(5.998, 6.002)$$

We are 95% confident that the true mean weight of all the Fidget Widgets produced is in the interval, (5.998, 6.002). Since 5 kg is not in this interval, 5 kg is not a plausible guess for the true mean mass of the Fidget Widgets.