

Show **all** of your work on this assignment and answer each question fully in the given context.

**If you cannot submit your homework in the class, you can drop it at my office door in 3220 Snedecore Hall by Thursday at 03:30 PM.**

Please staple your assignment and write your name !

1. [Ch. 5.1, Exercise 9, pg. 244] Transmission line interruptions in a telecommunication network occur at an average rate of one per day.

(a) Use a Poisson distribution as a model for

$X$  = the number of interruptions in the next five-day work week

Now, precisely specify the probability distribution.[5pts]

(b) Find  $P[X = 0]$ [5pts]

(c) Now consider the random variable

$Y$  = the number of work weeks in the next four in which there are no interruptions

What is a reasonable probability model for  $Y$ ?[5pts]

hint:i.e. precisely specify the probability distribution.

(d) Find  $P[Y = 2]$ .[5pts]

2. Suppose a standup comedian plans to give a total of  $n = 5$  jokes in an entire 2-hour performance. Call a joke a success if at least one audience member laughs. If no audience member laughs, the joke is a failure.

Assume that all the jokes are equally funny, with  $p = P(\text{success}) = 0.2$ . Let  $X$  be the random variable associated with the number of successful jokes out of the total 5.

(a) Precisely state the distribution of  $X$ , giving the values of any parameters necessary.[2pts]

(b) Calculate the probability that the whole night is a failure. i.e. find the  $P(\text{no success})$ . [5pts]

(c) Calculate the probability that the comedian tells at least 4 successful jokes.[5pts]

(d) Calculate the expected number of successful jokes.[5pts]

(e) Calculate the standard deviation of the successful jokes.[5pts]

3. The number of computer shutdowns during any month has a Poisson distribution, averaging 0.25 shutdowns per month.

(a) What is the probability of at least 2 computer shutdowns during the next year?[5pts]

(b) What is the probability of at most 2 computer shutdowns during the next 6 month? [5pts]

(c) What is the variance of the number of computer shutdowns during the next year? [2pts]

4. Suppose that  $X$  is a random variable with probability density function

$$f(x) = \begin{cases} c \cdot x^2 & -2 \leq x \leq 2 \\ 0 & o.w. \end{cases}$$

- (a) Find the value of  $c$  that makes  $f(x)$  a valid probability density function.[5 pts]  
(b) Find the CDF of the random variable  $X$ . [5 pts]  
(c) What is  $P(|X| \geq -1)$  [5 pts].  
(d) Find the expected value of  $X$ . [ 5 pts]
5. Consider a continuously distributed random variable,  $W$ , with a probability density function given by

$$f(w) = \begin{cases} \frac{1}{5(1-e^{-2})} e^{-w/5} & 0 \leq w \leq 10 \\ 0 & \text{otherwise} \end{cases}$$

- (a) Show that the function  $f(w)$  is a valid probability density function (i.e., show that (i)  $f(w)$  is non-negative and (ii)  $\int_{-\infty}^{\infty} f(w)dw = 1$ ). [5 pts]  
(b) Find  $P(W \leq 2)$  [5 pts]  
(c) Find  $P(2 \leq W \leq 10)$  [5 pts]  
(d) Find  $E(W)$  [5 pts]
6. [Ch. 5.2, Exercise 1, pg. 263] The random number generator supplied on a calculator is not terribly well chosen, in that values it generates are not adequately described by a uniform distribution on the interval  $(0, 1)$ . Suppose instead that a probability density

$$f(x) = \begin{cases} k(5 - x) & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

is a more appropriate model for  $X =$  the next value produced by this random number generator.

- (a) Find the value of  $k$ . [5pts]  
(b) Evaluate  $P[.25 < X < .75]$  [5pts]  
(c) Compute the cumulative probability distribution function for  $X$ ,  $F(x)$ . [5pts]  
(d) Calculate  $E(X)$  and the standard deviation of  $X$ . [6pts]

[Total: 115 pts]